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An acoustic metamaterial with space-time modulated density[a)]

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ABSTRACT:
Space-time modulation opens the door for unprecedented wave behavior control, such as nonreciprocal wave manipulation. Here is proposed a one-dimensional space-time modulated membrane system aiming to realize a kind of acoustic metamaterial with space-time modulated effective density. Three different approaches, namely, the effective medium method, transfer matrix method, and time-domain simulation, are applied to analyze the acoustic response of the system under a monochromatic incidence. Results show that the proposed metamaterial can support two different nonreciprocal acoustic functionalities, namely, unidirectional parametric amplification and parametric frequency conversion, when different modulation profiles are enforced. © *2024 Acoustical Society of America*.
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I. INTRODUCTION

Research on space-time modulated media dates back to the 1950s, when space-time modulation was first studied in transmission lines to break the time-reversal symmetry and realize nonreciprocal parametric amplification.[1–6] Compared to structures or materials that have spatially dependent properties, space-time modulation breaks the time-reversal symmetry of systems and then can break reciprocity, thus offering rich possibilities in wave manipulation and control, which are highly desired in a number of applications, such as sensing, imaging, and communication. However, despite the growing interest of nonreciprocal transmission in electromagnetics, acoustics, and many other physical fields, space-time modulation remained a scientific concept and had not been applied to practical dynamic wave control for a very long time.

Recently, thanks to the development of modulation techniques, space-time modulation has rejuvenated and attracted considerable attention through introduction into modern electromagnetics,[7–20] acoustics,[21–37] and mechanics.[38–42] For example, in airborne acoustics, by combining temporal modulation and spatial bias of several resonators, a lot of interesting and useful nonreciprocal devices have been proposed, such as acoustic isolators,[24,26,28] circulators,[21,22] amplifiers,[29,32] and metasurfaces.[30,31] Among these space-time modulated acoustic metamaterials, one of the most commonly utilized elements is the Helmholtz resonator with varying cavity.[24–27] Another promising and feasible structure is membrane utilizing the piezoelectric effect.[30]

In this work, we trace back to the space-time modulation of effective medium properties and focus on an acoustic metamaterial whose density varies in both space and time. As was derived in the previous research,[25] acoustic unidirectional parametric amplification and parametric frequency conversion would take place in such a space-time modulated metamaterial with a strategy for proper modulation of the density. However, how can we realize the space-time modulation of density? In this work, we'd like to give a feasible scheme. Membrane is commonly used in acoustic metamaterials to realize unusual densities[43] because of its resonance effect. Its resonance characteristic is not only closely related to the membrane itself, but also remarkably affected by the tension applied to it. So, we propose a membrane system and tune its effective density in a space-time varying fashion by controlling the membrane's surface tension. Three theoretical and numerical approaches, effective medium method, transfer matrix method, and time-domain simulation, are adopted to analyze the acoustic response of this membrane system from different perspectives. Two different modulation profiles are applied to the system to realize unidirectional parametric amplification and parametric frequency conversion functionalities, respectively.

II. METAMATERIAL REALIZATION OF SPACE-TIME MODULATED DENSITY

Consider a membrane system, as shown in Fig. 1(a). The system consists of a circular tube and a series of membranes that are edge-clamped and uniformly spaced in the tube. The radius of the tube and membranes is R, and the distance between two adjacent membranes is D. The tube is considered to be sound rigid to ensure one-dimensional (1D) wave propagation, and the background medium is air, whose

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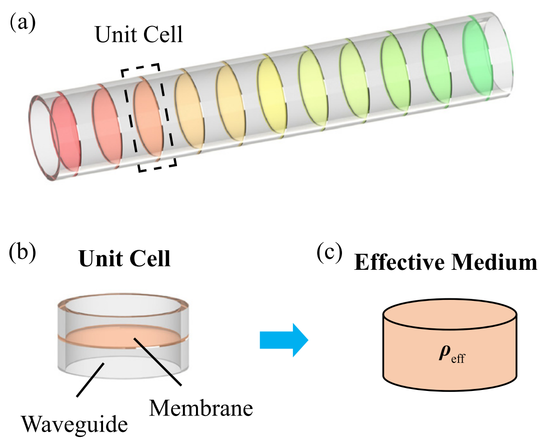

FIG. 1. (Color online) Acoustic metamaterial based on membranes. (a) Schematic of acoustic metamaterial. (b) Unit cell structure. (c) Effective medium of the unit cell.

density and sound speed are ρ_{air} and c_{air}, respectively. The membranes are made of silicone rubber with thickness d, density ρ_m, Young's modulus E_m, and shear modulus G_m.

This membrane system is a typical periodic structure and can be divided into a series of unit cells, as shown in Fig. 1(b). Due to the resonance of the membrane, each unit cell can be considered as medium of same volume with unusual effective density, as shown in Fig. 1(c), and the retrieval method of the effective density will be discussed in detail in Sec. III.

In order to modulate the membranes' resonance characteristics and thus modulate the effective density of the units, time-varying surface tensions in the form of $T_i = T_0[1 + m\cos(\Omega t - \phi_i)]^2$ are applied to the membranes. In the expression, i (=1, 2,…) is the ordinal number of the membrane. T_0 and m are the static surface tension and modulation depth. Ω and ϕ_i are the modulation frequency and phase, which control the modulation profile in time and space, respectively.

III. THEORETICAL AND NUMERICAL METHODS

In this work, we adopt three different methods to analyze the acoustic response of this space-time modulated system under a monochromatic incidence, namely, effective medium method, transfer matrix method and time-domain simulation. To simplify the analysis, loss and nonlinear effect are not taken into consideration in all three methods.

A. Effective medium method

The first method is called effective medium method, which predicts the acoustic effects under space-time modulation of density. In this method, the system is divided into a series of unit cells, as shown in Fig. 1(b), so that each unit cell can be treated as an effective medium whose interaction with incoming acoustic waves is dictated by its effective density, as shown in Fig. 1(c).

With previous preparation, we first get the effective acoustic parameters of each unit cell through a retrieval technique.[44] In this retrieval technique, we can use pressure acoustic frequency domain simulation to get the reflection coefficient R and transmission coefficient T for a plane wave normally incident on a single unit cell. The effective refractive index n and impedance z are obtained from R and T as

$$\begin{cases} n = \dfrac{\arccos\left(\dfrac{1 - R^2 + T^2}{2T}\right)}{\dfrac{\omega}{c_{\text{air}}} D}, \\ z = \sqrt{\dfrac{(1+R)^2 - T^2}{(1-R)^2 - T^2}}. \end{cases} \quad (1)$$

The effective density ρ and compressibility (inverse of bulk modulus) C are then calculated from n and z as

$$\begin{cases} \rho = \rho_{\text{air}} z n, \\ C = \dfrac{n}{\rho_{\text{air}} c_{\text{air}}^2 z}. \end{cases} \quad (2)$$

Owing to the space-time-varying surface tension applied to the membranes, the above membrane system can be turned into a space-time modulated medium whose density satisfies the form of $\rho(\omega) = \rho_0(\omega)[1 + m_\rho(\omega)\cos(\Omega t - \beta x)]$. ρ_0, m_ρ, Ω, and β are the static density (without modulation), modulation depth of density, modulation frequency, and phase gradient, respectively. All of these parameters can be obtained from the above retrieval technique through a parametric sweep study. The compressibility of this medium, on the other hand, can be assumed to remain constant, $C = C_0$, as the membranes' resonance has much less effect on it.

According to our previous work,[25] there will exist two acoustic modes in such a space-time modulated medium under a monochromatic incidence along the positive direction of the x axis (i.e., positive direction in the context). The frequencies of the incident mode and generated mode are ω_1 and ω_2, respectively. If the modulation frequency and phase gradient satisfy

$$\begin{cases} \Omega = \omega_1 + \omega_2, \\ \beta = k_1 + k_2, \end{cases} \quad (3)$$

the normalized acoustic pressure amplitude along the propagation direction in the medium will be

$$\begin{cases} P_1(x) = \cosh(\alpha x), \\ P_2(x) = \dfrac{z_2}{z_1}\sqrt{\dfrac{m_{\rho_1} k_1 \rho_1}{m_{\rho_2} k_2 \rho_2}}\sinh(\alpha x), \end{cases} \quad (4)$$

where

$$z_1 = z(\omega_1), \quad z_2 = z(\omega_2), \quad \rho_1 = \rho(\omega_1),$$
$$\rho_2 = \rho(\omega_2), \quad m_{\rho_1} = m_\rho(\omega_1), \quad m_{\rho_2} = m_\rho(\omega_2),$$
$$\alpha = \frac{\sqrt{m_{\rho_1} m_{\rho_2} k_1 k_2}}{4}.$$

Note the weak modulation assumption that $P_1(x)$ and $P_2(x)$ are slowly varying: Thus, $\partial^2 P_1(x)/\partial x^2$ and $\partial^2 P_2(x)/\partial x^2$ are negligible. From Eq. (4), we can see that the amplitudes of both modes are growing exponentially. While if the wave is incident along the negative direction of x axis (i.e., negative direction in the context), it will just pass through as in the unmodulated membrane system. This unusual effect is called unidirectional parametric amplification.

In addition, if the modulation frequency and phase gradient satisfy

$$\begin{cases} \Omega = \omega_1 - \omega_2, \\ \beta = k_1 - k_2, \end{cases} \tag{5}$$

the normalized acoustic pressure amplitude along the propagation direction in the medium will be

$$\begin{cases} P_1(x) = |\cos(\alpha x)|, \\ P_2(x) = \frac{z_2}{z_1}\sqrt{\frac{m_{\rho_1} k_1 \rho_1}{m_{\rho_2} k_2 \rho_2}}|\sin(\alpha x)|. \end{cases} \tag{6}$$

From Eq. (6), we can see that the amplitudes of two modes are periodically varying, which is called parametric frequency conversion.

B. Transfer matrix method

In our previous work, we've found that a series of Floquet components would be generated when a monochromatic acoustic wave is incident into the space-time modulated system.[26–28] The frequency of each harmonic is $\omega_n = \omega + n\Omega$, while $n = \ldots, -2, -1, 0, +1, +2, \ldots$ represents the order of harmonics. In the above effective medium method, an important assumption is that there will only exist two acoustic modes in such a space-time modulated medium under a monochromatic incidence, which means only the 0th incident mode and the -1st order generated harmonics are taken into consideration. This assumption makes the solution to the wave equation of this space-time modulated medium simple and helps us predict the unidirectional parametric amplification and parametric frequency conversion effects. However, ignoring of the high-order harmonics will make the prediction inaccurate or even wrong. So we introduce the second method, i.e., transfer matrix method, which provides fast and accurate calculation of general space-time-varying system and allows the investigation of high-order modes.[27]

In the transfer matrix method, our membrane system is divided into two types of acoustic elements, membrane and waveguide. Each element can be represented by a transfer matrix, and the corresponding derivation is completed in our previous work.[29] Here, we give the expression directly. The equivalent impedance of the modulated membrane is

$$Z_m(\omega) = -j\omega\rho_m d\left[J_0\left(\frac{\omega R}{c_m}\right)\right]\Big/\left[J_2\left(\frac{\omega R}{c_m}\right)\right].$$

J_0 and J_2 are the 0th order and 2nd order Bessel functions of the first kind. Under weak modulation ($m \le 0.1$ in this space-time modulated system), we can expand the impedance at c_{m0} as

$$Z_m(\omega) = Z_{m0}(\omega) + mZ_{v0}(\omega)\cos(\Omega t - \phi), \tag{7}$$

where

$$Z_{m0} = Z_m|_{c_m = c_{m0}},\ Z_{v0} = c_{m0}\frac{\partial Z_m}{\partial c_m}\Big|_{c_m = c_{m0}}$$

and

$$c_{m0} = \sqrt{\frac{T_0}{\rho_m d}}.$$

The modulated impedance can then be represented by a transfer matrix, namely,

$$M_m = \begin{bmatrix} & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 1 & Z_{m0}^{n-1} & 0 & \frac{mZ_{v0}^n}{2}e^{j\phi} & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & \frac{mZ_{v0}^{n-1}}{2}e^{-j\phi} & 1 & Z_{m0}^n & 0 & \frac{mZ_{v0}^{n+1}}{2}e^{j\phi} & \cdots \\ \cdots & 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \frac{mZ_{v0}^n}{2}e^{-j\phi} & 1 & Z_{m0}^{n+1} & \cdots \\ \cdots & 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix}, \tag{8}$$

where $Z_{m0}^n = Z_{m0}(\omega_n)$ and $Z_{v0}^n = Z_{v0}(\omega_n)$. In addition, the transfer matrix of the waveguide can be written directly,

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$$M_w = \begin{bmatrix} & \vdots & \vdots & \vdots & \\ \cdots & M_w^{n-1} & 0 & 0 & \cdots \\ \cdots & 0 & M_w^n & 0 & \cdots \\ \cdots & 0 & 0 & M_w^{n+1} & \cdots \\ & \vdots & \vdots & \vdots & \end{bmatrix}, \tag{9}$$

where

$$M_w^n = \begin{bmatrix} \cos(k_n D) & j z_{air} \sin(k_n D) \\ \dfrac{j}{z_{air}} \sin(k_n D) & \cos(k_n D) \end{bmatrix}. \tag{10}$$

Here, $z_{\mathrm{air}} = \rho_{\mathrm{air}} c_{\mathrm{air}}$ is the characteristic impedance of air, $k_n = \omega_n / c_{\mathrm{air}}$ is the wave number of the nth order wave, and D is the length of each section of waveguide.

With the transfer matrix for the membrane and waveguide, the transfer matrix for the entire system can be calculated by multiplying the transfer matrices for all components. Hence, the acoustic response of the system can be obtained. With the transfer matrix method, we can, in principle, take all orders of harmonics into account. However, in practice, the matrix shall be truncated to account only for the orders that are non-negligible.

C. Time-domain simulation

The third method, unlike the frequency domain analysis employed in the aforementioned two methods, is a numerical approach conducted in the time domain. It emulates the experimental setup and serves to validate the anticipated unidirectional parametric amplification and parametric frequency conversion. This time-domain simulation is implemented in COMSOL and adopts a MULTIPHYSICS setup where both pressure acoustic module and membrane module are used.

First, a 1D model of the membrane system is built up, which is divided into three sections. The most important section is the modulated section, where the space-time modulated initial stress $T_i = T_0[1 + m\cos(\Omega t - \beta z)]^2$ is applied to the membranes. The other two sections before and after the modulated section are called upstream and downstream, respectively, where the membranes' initial stress is constant T_0. In the unidirectional parametric amplification case, the length of the modulation section is 0.25 m, containing 50 space-time modulated unit cells. In the parametric frequency case, the modulation length is 0.5 m, which contains 100 unit cells instead. The length of the upstream and downstream is 1 m, both containing 200 unmodulated unit cells.

Then, a time-dependent study is conducted and a sinusoidal wave is incident from the upstream. The element size is predefined normal, and the time step is 1×10^{-5} s. Probes are placed at the interface of adjacent unit cells in the modulation section to record the corresponding pressure. In the unidirectional parametric amplification case, we simulate the wave incident from both positive and negative directions to demonstrate the distinct interactions with space-time modulation, where the negative incident direction was simulated by switching the sign of the modulation wave number to $-\beta$. In the parametric frequency conversion case, only the positive direction propagation is interested.

Finally, acoustic signals recorded by the probes are analyzed. In the unidirectional parametric amplification case, a total of 51 probes are utilized, resulting in the recording of 51 acoustic signals. To investigate the steady-state wave behavior, each signal's waveform is captured from 0.08 to 0.12 s and subsequently subjected to Fourier transform for spectrum analysis. In the parametric frequency conversion case, there are 101 probes, and their respective waveforms from 0.10 to 0.14 s are processed accordingly. By conducting spectrum analysis on each detection point within the modulation section, we can ascertain variations in each of the frequency components along the propagation direction and thereby demonstrate the unidirectional parametric amplification and parametric frequency conversion.

IV. RESULTS AND DISCUSSION

In this work, we realize the above density modulated media via membrane system, as shown in Fig. 1. The parameters in our system are given as follows: $\rho_{\mathrm{air}} = 1.21$ kg/m^3, $c_{\mathrm{air}} = 343$ m/s, $\rho_m = 1300$ kg/m^3, $E_m = 117.5$ kPa, $G_m = 40$ kPa, $R = 7.5$ mm, $d = 0.2$ mm, $D = 5$ mm, $T_0 = 1$ MPa $\times d$, and $m = 0.05$. To start with, the band structure of the unit cell is first calculated, as shown in Fig. 2.

A. Unidirectional parametric amplification

According to the effective medium method, to realize the unidirectional parametric amplification, we should design the modulation parameters Ω and k to satisfy Eq. (3). In this case, the two frequency components of interest are $\omega_1 = 2\pi \times 2050$ rad/s for the incident wave and $\omega_2 = 2\pi \times 1550$ rad/s for the generated wave. The corresponding wave numbers are $k_1 = 241.6$ rad/m and $k_2 = 98.4$ rad/m, as shown in Fig. 2. So, we can get the modulation frequency and phase gradient of the system as $\Omega = 2\pi \times 3600$ rad/s and $\beta = 340.0$ rad/m. Then, the effective density and compressibility of the modulated unit cell are computed through the retrieval technique, and the results are shown in Fig. 3. It can be observed that with such kind

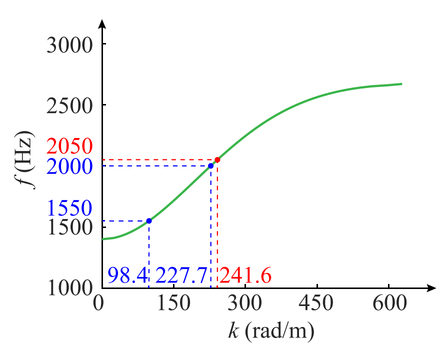

FIG. 2. (Color online) Dispersion curve of the unit cell.

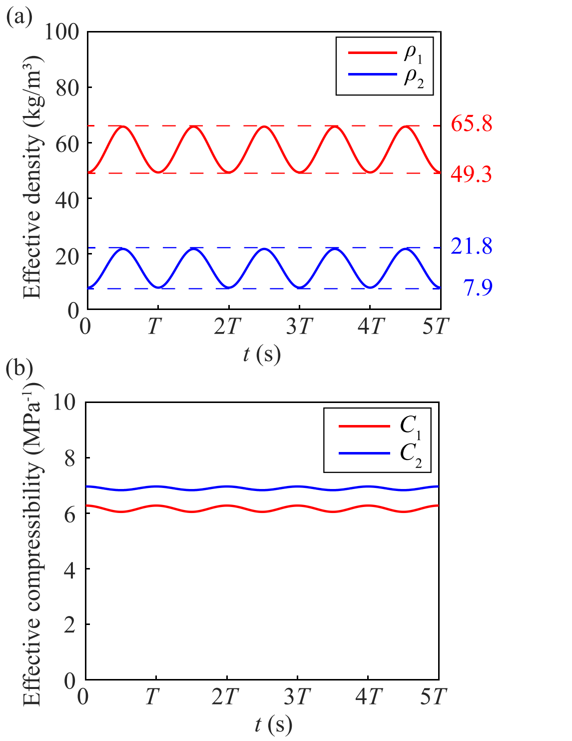

FIG. 3. (Color online) (a) Effective density and (b) compressibility of the unit cell under harmonic modulation (unidirectional parametric amplification). The red line indicates effective parameters for mode 1 ($f_1 = 2050$ Hz) and the blue line indicates effective parameters for mode 2 ($f_2 = 1550$ Hz).

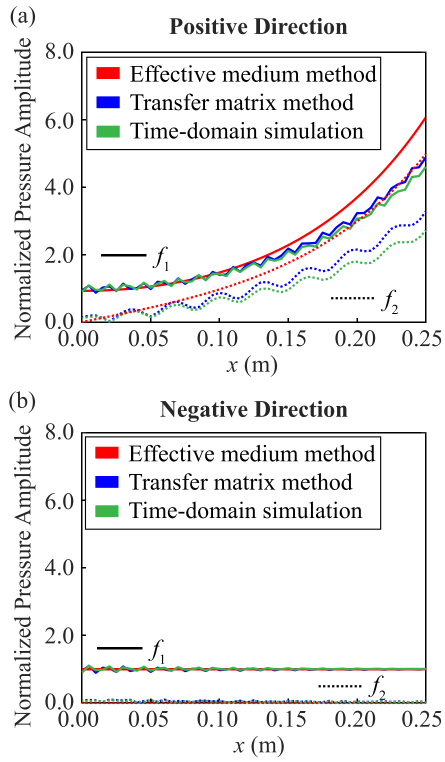

FIG. 4. (Color online) Unidirectional parametric amplification effect along the propagation distance (a) in the positive direction and (b) in the negative direction. Solid and dashed lines represent modes 1 and 2, respectively. Red, blue, and green represent three different methods: effective medium method, transfer matrix method, and time-domain simulation, respectively.

of time-varying surface tension, the effective density is sinusoidally modulated and the modulation period $T = 2\pi/\Omega$. We can get the static density and modulation depth for two modes as $\rho_{10} = (65.8 + 49.3)/2 = 57.55$ kg/m^3, $\rho_{20} = (21.8 + 7.9)/2 = 14.85$ kg/m^3, $m_{\rho 1} = (65.8 - 49.3)/(2 \times 57.55) = 0.143$, and $m_{\rho 2} = (21.8 - 7.9)/(2 \times 14.85) = 0.468$, respectively. In contrast, the modulation of compressibility is much weaker and negligible, which provides support for the assumption of constant compressibility in this method. Note the slight difference between C_1 and C_2 is mainly attributed to frequency dependence of membrane resonance, where $C_1 = C(\omega_1)$ and $C_2 = C(\omega_2)$. Now we have all of the parameters needed for the effective medium method to predict the acoustic response in this space-time modulated medium. The acoustic pressure amplitude variation along the propagation in both the positive and negative directions is illustrated in red in Fig. 4.

To verify the prediction of the effective medium method and demonstrate the unidirectional parametric amplification in this space-time modulated membrane system, we use the transfer matrix method and time-domain simulation to calculate the acoustic response in frequency and time domain, respectively. Results are illustrated in blue and green in Fig. 4, respectively. Note that the pressure amplitude from the time-domain simulation is normalized to compare with the results from the other two methods. As expected, in the positive direction, obvious exponential growth of both frequency components is observed. In the negative direction, only the incident frequency component exists, and its pressure amplitude remains unchanged. It is also observed that the results from transfer matrix method and time-domain simulation agree well with each other, while the growth rate from the effective medium method is a bit larger. This deviation can be attributed to the imperfect discretization of the acoustic metamaterial by a finite number of unit cells, which also results in the pressure amplitude oscillation.

B. Parametric frequency conversion

Now we come to the parametric frequency conversion case. All parameters of the membrane system stay unchanged, except the modulation frequency Ω and phase gradient β, which should satisfy Eq. (5). In this case, the two modes of interest are $\omega_1 = 2\pi \times 2050$ rad/s for the incident wave and $\omega_2 = 2\pi \times 2000$ rad/s for the generated wave. The corresponding wave numbers are $k_1 = 241.6$ rad/m and $k_2 = 227.7$ rad/m, marked in Fig. 2. So, we can get the modulation frequency and phase gradient of the system as $\Omega = 2\pi \times 50$ rad/s and $\beta = 13.9$ rad/m. Similarly, the effective density and compressibility of the unit cell are calculated and illustrated in Fig. 5. We can get the static density and modulation depth for two modes as $\rho_{10} = (65.8 + 49.3)/2 = 57.55$ kg/m^3, $\rho_{20} = (60.5 + 45.1)/2 = 52.8$ kg/m^3, $m_{\rho 1} = (65.8 - 49.3)/(2 \times 57.55) = 0.143$, and $m_{\rho 2} = (60.5 - 45.1)/(2 \times 52.8) = 0.146$, respectively. The modulation of compressibility is also much

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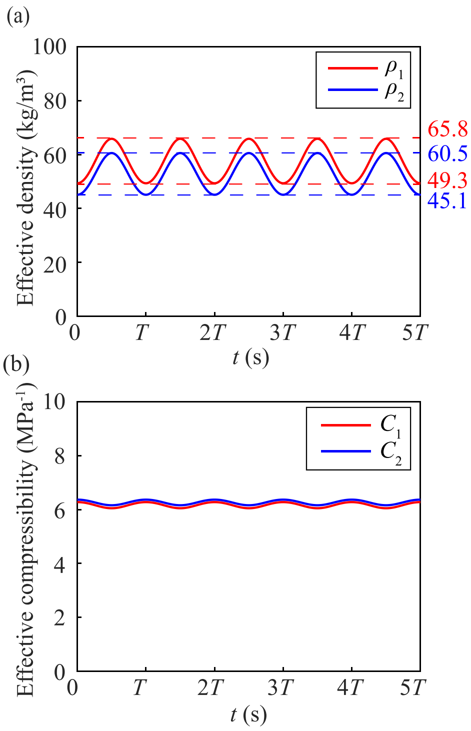

FIG. 5. (Color online) (a) Effective density and (b) compressibility of the unit cell under harmonic modulation (parametric frequency conversion). The red line indicates effective parameters for mode 1 ($f_1 = 2050$ Hz) and the blue line effective parameters for mode 2 ($f_2 = 2000$ Hz).

weaker and negligible. Based on the effective medium method with above parameters, we can depict the acoustic pressure amplitude along the propagation direction, as shown in Fig. 6(a). It is observed that the amplitudes of two modes are periodically varying, which means the modes are transferred back and forth along the propagation.

Then, we would like to verify this frequency conversion effect in the space-time modulated membrane system using the transfer matrix method and time-domain simulation in frequency and time domain, respectively. However, we've found that the acoustic response of the membrane system obtained from both two methods is totally different from the effective medium method's prediction, as shown in Figs. 6(b) and 6(c). In the meantime, results from the transfer matrix method and time-domain simulation agree pretty well with each other. In Figs. 6(b) and 6(c), it is observed that there are many other frequency components existing in this space-time modulated membrane system, rather than only two modes predicted by the effective medium method. Results show that in this space-time modulated membrane system, there does exist a frequency conversion effect, but this frequency conversion involves a lot of high-order harmonics and the acoustic energy gradually transfers to the higher-order harmonics as the wave propagates forward. Recall that an important assumption in the effective medium method is that there are only two modes existing in the space-time modulated medium. These results indicate the failure of this primary assumption. Yet, why? Why does the effective medium method work in the unidirectional parametric amplification while failing in the parametric frequency conversion?

C. Analysis of mode coupling and conversion

The main reason for the effective medium method's failure in the parametric frequency conversion case is the fundamental assumption that there are only two acoustic modes converting in the system. Results from the other two methods in Figs. 6(b) and 6(c) both show that there are a lot of higher-order harmonics in the system, whose proportion cannot be neglected. These higher-order harmonics' generation and conversion can also be explained through Fig. 7. The green solid line represents all of the acoustic modes supported by the system, and the red point represents the incident mode. If there is no modulation in this membrane system, only the incident mode will transmit in the system and no other modes will appear. When a sinusoidal modulation is applied to the system, it offers the possibility for the generation of some certain modes, $\omega_n = \omega_0 + n\Omega$ ($n = \ldots, -2, -1, 0, 1, 2, \ldots$), represented by a magenta dashed line in Fig. 7. In principle, mode conversion can occur if the modes coexist on the dispersion curve (green solid line) and the modulation relation (magenta dashed line). An incident mode can be converted to the other mode as both of them are supported in this space-time modulated system.

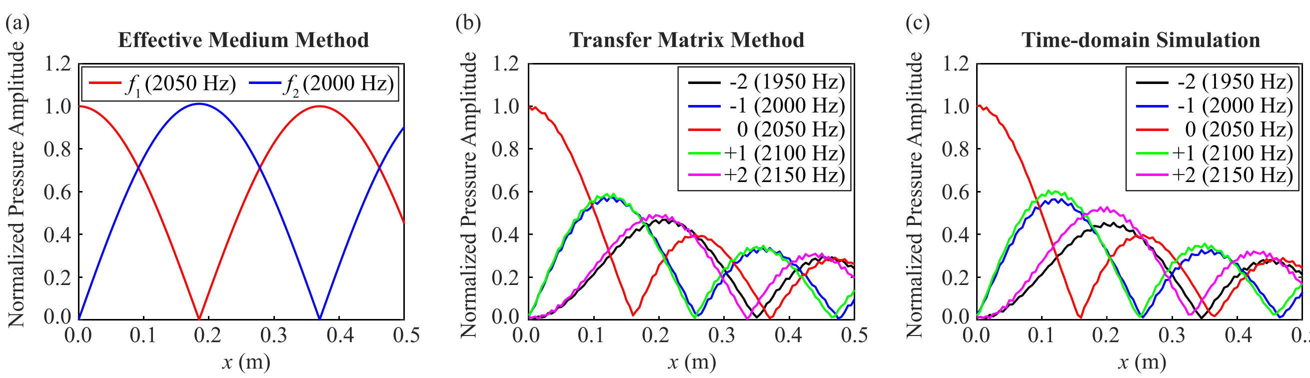

FIG. 6. (Color online) Parametric frequency conversion effect along the propagation direction calculated by three different methods: (a) the effective medium method. (b) transfer matrix method, and (c) time-domain simulation. The red line indicates the 0th fundamental frequency (mode 1), the blue line indicates the -1st harmonic (mode 2), the green line indicates the $+1$st harmonic, the black line indicates the -2nd harmonic, and the magenta line indicates the $+2$nd harmonic.

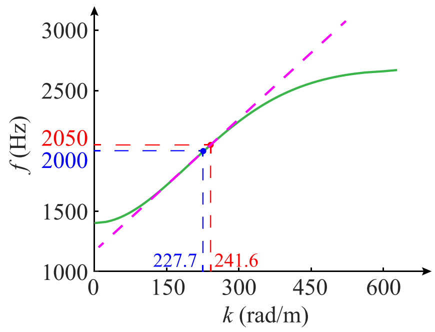

FIG. 7. (Color online) Dispersion curve and mode coupling of the metamaterial based on membrane (parametric frequency conversion). The red and blue dots represent modes 1 and 2 in the system, respectively.

Observing Fig. 7, although there are only two modes that strictly meet the above condition, other harmonics are also very close to the dispersion curve, which means they are also inter-convertible even if it is not ideal conversion. While in the unidirectional parametric amplification case, as shown in Fig. 8, the dispersion curve (green solid line) and the modulation relation (magenta dashed line) are far apart from each other except for the two points of intersection (incident mode f_1 and generated mode f_2), which means no other harmonics are coupled in. Therefore, in this case the fundamental assumption that there are only two acoustic modes converting in the system is valid.

V. CONCLUSION

To conclude, a mathematical model of highly idealized membrane system is proposed, completely ignoring loss and nonlinearity. With space-time modulated surface tension applied to membranes, this system can be considered as an acoustic metamaterial with space-time modulated density. In theory (effective medium method), two special acoustic phenomena, unidirectional parametric amplification and parametric frequency conversion, would occur in the metamaterial under two specific modulation profiles. Two different approaches, transfer matrix method and time-domain simulation, are applied to analyze the acoustic response of the system

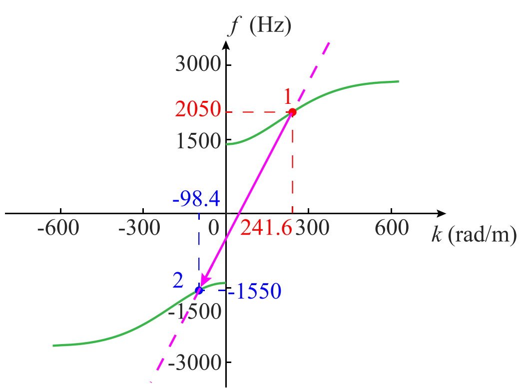

FIG. 8. (Color online) Dispersion curve and mode coupling of the metamaterial based on membrane (unidirectional parametric amplification). The red and blue dots represent modes 1 and 2 in the system, respectively.

from the frequency and time domain, respectively. Calculations indicate that this space-time modulated membrane system does support unidirectional parametric amplification and parametric frequency conversion in theory. However, the acoustic response of the membrane system obtained from the transfer matrix method and time-domain simulation are not entirely the same as those predicted from the effective medium method, especially for the parametric frequency conversion case. The difference in results between the effective medium method and the other two methods is attributed to the imperfect approximation of the membrane system as a density-modulated metamaterial and some ideal assumption in the effective medium method's derivation, which can be explained through the model coupling and conversion analysis.

In this work, we mainly communicate an interesting idea of effective density space-time modulation using tension-controlled membranes and discuss the interesting acoustic phenomena occurring in this space-time modulated metamaterial. The main limitation of the model proposed in this work lies in its failure to account for loss and nonlinearity. Further, we only analyze the unidirectional parametric amplification and parametric frequency conversion under two specific modulation profiles. As this space-time modulated membrane system offers many degrees of freedom and supports many more modulation profiles, it can go far beyond these two primary functionalities by applying other modulation strategies. What is more, it is technically difficult to control membranes' surface tension directly, especially in space and time simultaneously. A feasible experiment scheme is to use a piezoelectric membrane and convert the tension's control into electrical control. The corresponding relation between the membrane's effective surface tension and the external voltage applied needs further study.

In brief, we hope that the proposed membrane-based space-time modulated model can serve as the foundation for various research that require both spatial and temporal control of the effective density.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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